VOTES AND SEATS: INDEXES, MEASURES AND THEIR APPLICATION

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Abstract: The paper presents the most known indexes used to describe and analyze electoral and legislative fragmentation/concentration. Based on literature review, the research examines the concepts and formulas of the indexes and presents examples of application. Besides, it discusses peculiarities of the Brazilian case, like mallaportionment and electoral coalitions in proportional systems. To know the indexes and their correct use are important to knowledge accumulation and generation of standard basis to promote comparative analysis. The main finding of the paper is that each index has a specific purpose, they can supply just partial information to create a broad picture of the analyzed system.

Keywords: index; electoral fragmentation; legislative fragmentation; elections; legislature.

1 Introduction

Elections distribute votes and seats, and thus are central elements of democratic-representative systems (MANIN, 1997). The electoral process, with its practices and institutions, structures representation and gives functionality to the political system. The analysis and measurement of electoral performance and party conformation at assemblies are important and should follow recognized methodology, which implies being tested, understood and shared by the scientific community. This article presents the main indexes that measure electoral performance and distribution of legislative seats, exposes and exemplifies its calculation methodologies and discusses its application to the singularities of the Brazilian case.

The electoral systems available for the formation of the Legislative Branch are the proportional, the majority and the mixed (NICOLAU, 2012). They differ mainly in how they turn citizens' votes into parliamentary seats. The proportional system seeks to make the most of the voters' manifestation in the ballet box, "wasting" the minimum of votes and pluralizing the representation, while the majority system is concerned with the selection of the most voted, relegating to oblivion a significant part of the votes – those conferred to defeated candidates. Proportional systems stimulate diversity, and majority systems, the formation of more cohesive and homogeneous governments (LIJPHART, 1999; NICOLAU, 2012). Mixed systems try to

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combine the strengths of both fundamental electoral systems.

Studies on the relationship between electoral and party systems are not new in Political Science, on the contrary. Seminal studies such as those of Duverger (1980) [1951] and Rae (1967) have continued an important line of analysis, ranging from the discussion of electoral systems in the nineteenth century. Also, electoral and partisan areas and their relations are central themes of political engineering proposals (for example, Htun and Powell Junior, 2013).

In what has become known as the "Duverger’s Law" – in a non-strict sense of scientific law - the electoral system influences the configuration of the party system, leading it to fragmentation: majority electoral systems of one ballot encourages the creation of two-party systems, while two-rounds (run-off voting) major systems and proportional systems, the multi-party systems (DUVERGER, 1980). Rae’s work (1967) complements that of Duverger (1980), since the first advances in the quantitative dimension of the political implications of electoral systems, by creating several indexes that show the degree of competitiveness of party systems. Later studies, such as Laakso and Taagepera (1979), Pedersen (1979) and Gallagher (1991), walk on the previously open path.

One of the factors responsible for the evolution and deepening of electoral and partisan studies was the development of indexes, which represent "[...] the final aggregate value of a whole calculation procedure where indicators are used as variables that the compose" (SICHE et al., 2007, p. 139). The creation of indexes enabled not only a more detailed knowledge of electoral and party realities, but also stimulated comparative analyses between elections in the same country and evaluations between electoral systems and elections in several countries. Fragmentation or concentration of votes and seats constitute a central dimension of representative political systems, and quantifying them is basic scientific endeavor, a necessary step to dialogue with the knowledge of the area, replicate studies and methodologies, test results and advance to new theories.

This article aims to present classic measures of fragmentation, concentration, and proportionality of votes and seats (RAE, 1967; LAAKSO and TAAGEPERA, 1979; PEDERSEN, 1979; GALLAGHER, 1991). It is intended to allow the reader a didactic apprehension of the construction of the measures and their meaning.

The article is divided in this introduction, in a section for the presentation and discussion of the contents and in the conclusion.

2 Electoral and Party Fragmentation Measures

The measurement allowed by the indexes used in this article has an appropriate sense when evaluating the collegiate political bodies, since it is where we can speak of fragmentation of votes and of fragmentation of seats[4]. In the case of a single elected, as in the choice of the President of the Republic, there is fragmentation only of votes, since the representation, that is, "the seat", is conferred to only one agent. Thus, when one speaks of the number of seats, one is referring to collegiate bodies, representative assemblies of all types.

In legislative elections, there are systems that operate with the vote in the candidate and others with the vote in the party, however, the measures listed here are always based on the participation of the political party. Adding votes and seats by parties makes sense for at least two reasons. The first is that parties, although they have undergone several transformations over the last two centuries, remain as the central organizational structures in the elections dispute (AMARAL, 2013). he second refers to the functioning of the parliamentary arena, in which the whole logic of action is collective. In this space, the parties hold a central position in the coordination of political actions, which applies to Brazil (FIGUEIREDO and LIMONGI, 1999)[5].

The measure constructs used are elaborated from two basic elements: the participation of the party in the total of votes and in the total of seats. Several measures analyze the distribution of votes and seats, and a logical division could separate them as follows:

i) measures that use only one element (votes received by the party or seats intended for the party);

ii) measures involving two elements (votes and seats);

iii) measures that take only one element in several elections (votes or seats at times t_0, t_1, etc.)[6].

The basic data worked by Rae (1967), a pioneer in the quantitative studies on electoral or party concentration or fragmentation, are total votes per party and total seats per party. The total number of votes (seats) received by a party in relation to the total votes (seats) received by all competitors in an election indicates their (parliamentary) electoral strength, that is, the proportional strength that a party possesses vis-à-vis voters (the other parties in the Legislative).

We must consider some peculiar and significant issues for the Brazilian case, which make the application of general indexes require precautions. In his analysis, Rae (1967) starts from the European and North American experiences and does not problematize the basic democratic assumption of "one man, one vote". Consider, for example, the British case, where all districts seek the same number of voters (Nicolau, 2012). Adding votes and seats throughout the system does not bring major complications. In the Brazilian case, the assumption of "one man, one vote"

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4 The term fragmentation can be taken in a specific or generic sense. In the specialized literature (NICOLAU, 2005; SANTOS, 2004), fragmentation has a specific meaning: the result of the division of the F index by maximum fragmentation ($F_{maximun}$). Already fragmentation in the generic sense corresponds to the idea of a system divided into several constituent elements. The general meaning is present both in Rae's work (1967) and in the other two Brazilian authors cited in this note. Fragmentation, in this article, is taken broadly and fractionalization equates to Rae's F index, characterized by a specific definition.

5 One could imagine aggregations by electoral coalitions or legislative coalitions. While this is possible, it is not the path taken in this article. Understanding aggregation by parties allows the reader to learn the mechanism and, if desired, to make other aggregations.

6 It logically follows that there may be analysis of two elements in time-series, but no manifestations of this nature were identified in the consulted literature.

does not apply to the federal deputy elections. Some states are underrepresented[7], such as São Paulo, and others are overrepresented, such as the scarcely populated states of the North, due to the constitutional criterion of maximum and minimum numbers of deputies (70 and 8, respectively). Adding votes from all over the territory brings a bias, insofar as the parties do not have a homogeneous distribution of votes throughout the country.

As an example, in the 2014 elections in the state of Roraima, 29,762 votes were required to elect a deputy, already in São Paulo, 303,738 (BRASIL, TSE, 2014). Imagine a strong party in Roraima and absent in São Paulo, and another with an opposite performance, strong in São Paulo and absent in Roraima. Even with a significant difference of votes in the national calculation, both may have elected the same number of parliamentarians. When votes from different states are aggregated, votes are added with different weights in terms of the production of legislative seats.

The second issue peculiar to the Brazilian context refers to the system of electoral coalitions. In Brazil, found the electoral and partisan quotients, the number of seats distributed to each coalition is defined. The most voted, in a single list of the coalition, are the chosen ones. In the division of intra-colligation seats, the total votes of each party are not respected, only the order of the most voted candidates. In other countries, there are coalitions that work only to overcome the barrier clause, with the seats being distributed proportionally to the number of votes each party contributed to the coalition (NICOLAU, 2012). In Brazil, some parties may benefit disproportionately from the coalition, taking more seats than votes, or suffering the opposite effect, with fewer seats than votes (DIAP, 2014, 138-9). There is a potential distortion between participation in the votes and in the seats, supported by the institute of the coalitions, according to the current legislation.

Once these observations have been made, the indexes will be presented.

2.1 Measures with Votes

Following the notation developed by Rae (1967), the measures related to the votes received by the parties are always indicated by letter (s) capitalized (s) accompanied (s) by "e" subscribed, which means electoral parties.

2.1.A. Number of parties (N_e), of Rae (1967)[8]: is the number of parties that received votes in an election. It shows only the number of parties that competed in the election and received votes, without concerning about the electoral strength of each party. In the two situations in Table 1, $N_e = 3$.

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7 The expression *mallaportionment* is used to define the phenomenon, the existence of electoral districts with different proportions of voters for the deputies.

8 Some indexes' denominations offer direct translations. Others, because they became somewhat confusing, have been modified to make themselves clear. Nor have all translations been compared with the frequent terms in the national literature. However, this should not lead to significant loss, because the main indexes have canonical translations.

Table 1 – Example of number of parties (N_e)[9]

	Situation 1 Votes (V)	Situation 2 Votes (V)
Party A	0,10	0,35
Party B	0,30	0,33
Party C	0,60	0,32
Σ	1,00	1,00

Source: Authors' elaboration.

2.1.B. Electoral participation of the strongest party (P_e), by Rae (1967): is the proportion of votes received by the most voted party. It measures the largest electoral force but does not aggregate information about the other competing parties. Table 2 shows that in situation 1, Party C received the highest percentage of votes, in the case 60%, while in situation 2, Party A obtained the highest percentage, 35%.

Table 2 – Example of the strongest party (P_e)

Situation 1	Situation 2
Party C = 0,60	Party A = 0,35
P_e = 0,60	P_e = 0,35

Source: Authors' elaboration.

2.1.C. Electoral participation of the two strongest parties (W_e), by Rae (1967): is the sum of the percentages of votes received by the two best-voted parties in the election. It shows how much the two best-placed parties in the dispute predominate over the others. However, it does not show the percentages of votes of each of them and the remaining parties.

We cannot say that W_e always show how close the system is to two-party system. A low value of W_e, 50%, for example, shows that the system does not have two parties holding the most votes, which distance the two-party system. However, a high value does not prove two-party system. W_e equal to 90%, for example, can be found in the case where the two largest parties have 45% of votes each, which constitutes a two-party system, or, among other hypotheses, that the largest is 80% and the second, 10% of the vote, something far from a two-party system.

9 We chose to preferentially make the decimal notation in the Tables and the percentage notation in the text.

Table 3 – Example of the two strongest parties (W_e)

Situation 1	Situation 2
$W_e = V_c + V_b$	$W_e = V_a + V_b$
$W_e = 0{,}60 + 0{,}30 = 0{,}90$	$W_e = 0{,}35 + 0{,}33 = 0{,}68$

Source: Authors' elaboration.

The indexes 2.1.A., 2.1.B. and 2.1.1.C are very basic and descriptive. The next index is more complex.

Fractionalization

"Fractionalization means division into various parts "(RAE, 1967, p. 54, our translation). This fractionalization considers: i) how many elements make up a system; and ii) the relative importance or size of each of the elements. Figure 1 shows three different situations. In A we have a non-fragmented system: it is composed of only one part and that part is the whole circle. In B there is a fractionalized system, divided into four equal parts. Already in C are also four parts, but one of them bigger and the other three smaller and equal. Visually it can be affirmed that B and C are more fractionalized than A but knowing to what extent they are fractionalized requires measurement by a criterion established for that purpose.

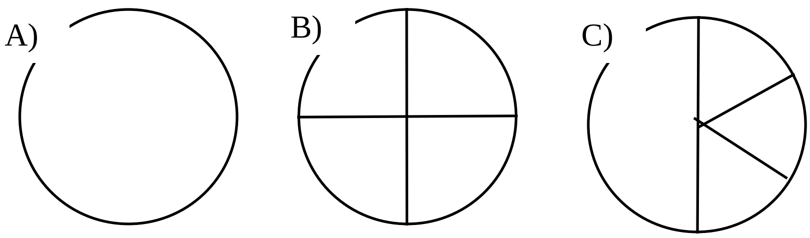

Figure 1: Fractionalization

Source: Authors' elaboration.

2.1.D. HH (Herfindal-Hirschman) index: originally created to evaluate **concentration** of companies in a market, the Herfindal-Hirschman index is the matrix of fractionalization indexes.

The HH index ranges from zero to 1, the higher the concentration the closer it is to the value 1. The HH index ranges from zero to 1, the higher the concentration the closer it is to the value 1. This is a probability calculation. Taking the electoral situation in Table 4, and considering situation 1, the probability that two voters have voted in party A is the chance of the first voter being party A (90%) and that of the second voter also (90%) (equal to 90% squared, $(0{,}90)^2$ By the same logic, the probability of two voters voting in party B is 6 percent squared ($(0{,}06)^2$) and that they have voted in party C is 4 percent squared ($(0{,}04)^2$). The probability of two voters voting in the same party, whatever the party, is the addition of the individual probabilities for each party $((0{,}9)^2 + (0{,}06)^2 + (0{,}04)^2)$.

The HH index can be expressed by:

$HH = \sum V_i^2$, where "V" is equal to the fraction of votes received by a party and "i" runs the parties from 1 to N, that is, all parties are considered.

The HH index for situation 1 of Table 4 is 0.8152. That is, the chance that two randomly chosen voters have voted in the same party is 81.52%.

2.1.E. Rae's (1967) fractionalization of votes (F_e) index: indicates the probability that two randomly chosen voters have voted in **different parties** in an election. The HH index expresses concentration and that of Rae, fractionalization. This is undoubtedly among the indexes elaborated by Rae, the most important and known. It is written as follows:

$F_e = 1 - (\sum V_i^2)$, where "V" and "i" are as defined in 2.1.D.

By subtracting from unit (1) the probability of two voters voting in the same party, has its complement, that is, the probability of two randomly chosen voters not having voted in the same party. The Rae F index is therefore a relatively simple extension of the HH index.

It is important to emphasize that Rae's mathematical construction has a concrete, intuitive sense, because if two voters are randomly drawn, the chance that they have not voted in the same party is equal to the value of F_e. This intuitive aspect is important and useful in interpretation of a measure.

Table 4 – Example of percentage of party votes and of the votes fractionalization index (F_e)

V	Situation 1	Situation 2
V_a	0,90	0,34
V_b	0,06	0,33
V_c	0,04	0,33
$\sum$	1,00	1,00
F_e	0,1848	0,6666

Source: Rae (1967, p. 54).

Table 4 shows two electoral situations, the second much more fractionalized than the first. Its values, for the measure F_e, are: in the first situation, 0.1848, and in the second, 0.6666. So, the probability of two randomly chosen voters voting in different parties is 18.48% in situation 1 and 66.66% in situation 2. Situation 1 illustrates little party fractionalization and approaches a single party system, because the votes are concentrated in a single party, although there are three junctions in this system. Situation 2 reflects a lot of fractionalization and reveals, in fact, a multi-party system of three parties, since all three received very similar percentages of votes.

A measure related to the F index is that of maximum fractionalization ($F_{maximum}$), that considered when, given several parties that obtained votes, they all reach identical portions of the electorate (Rae, 1967). In Table 4, with three parties, for both situations, the maximum fractionalization index is 66.67%, which derives from each party reaching approximately 33.33% of the votes. From the division of F effectively found by theorist $F_{maximun}$, one arrives at what is called fragmentation, according to footnote 1, a concept also worked on in several scientific studies (Santos, 2004).

Comparison between HH and

The Rae index F subtracts the HH index from 1. It is assumed that:

$F_e = 1 - HH$. Or, as already shown:

$F_e = 1 - (\sum V_i^2)$, where "V" and "i" are as defined in 2.1.D.

Table 5 shows a comparison between the average values of the F_e and HH indexes.

Table 5 – Average values of F_e and HH

	F_e (Rae)		
more concentrated	0	1	more fractionalized
	HH		
more fractionalized	0	1	more concentrated

Source: Authors' elaboration.

Nicolau (2005), mentioning the criticism of Sartori (1982), points out that the great problem of the F index, like that of HH, is that it potentiates the contribution of the major parties and decreases the importance of the minors. This is because the formula for calculating the indexes raises the proportional contribution of each party to the square, situation in which, the closer to zero the value, the lower its result when squared. For example, a hypothetical party α with 50% of the votes contributes to the calculation of the indexes with 0.25 (result of $0{,}5^2$). Already the party **Ω**, with half the size of the previous one, 25% of the votes, contributes with a value for the indexes of 0.0625. In other words, the party **Ω** is half the size of the party α but contributes only a quarter of its contribution to the F and HH indexes.

This observation, despite the apparent aggression to intuition, is the fruit of only a specific interpretation of the index. Santos (2004) points out that the original sense of probability in the F index is consistent, and therefore the interpretation of the indexes must be deepened. It is a probabilistic calculation, with specific meaning.

Although criticism deserves this statistical consideration, Nolte and Sánchez (2005) find that F_e presents more proportional results only when there are few parties in the system, between three and six. That is, the index behaves in a way that less strikes our intuition when working with a margin of three to six parties.

Table 6 – Examples of the HH index and the fractionalization index of votes (F_e)

V	Situation 1	Calculation V_i^2	Situation 2	Calculation V_i^2
V_a	0,45	0,45 x 0,45 = 0,2025	0,34	0,34 x 0,34 = 0,1156
V_b	0,29	0,29 x 0,29 = 0,0841	0,33	0,33 x 0,33 = 0,1089
V_c	0,21	0,21 x 0,21 = 0,0441	0,33	0,33 x 0,33 = 0,1089
V_d	0,05	0,05 x 0,05 = 0,0025	0,00	0,00
$\sum$	1,00	0,3322	1,00	0,3334
F_e		1 - 0,3322 = 0,6668		1 - 0,3334 = 0,6666

Source: Authors' elaboration.

A criticism that can be made to F_e, as well as to HH, is that different situations can present equivalent results (NICOLAU, 1997; SANTOS, 2004). As Table 6 illustrates, Situation 1 and Situation 2 indicate practically the same value of F_e. However, in the first case there is a four-party system with the largest of them almost reaching an absolute majority of votes received, while in the second hypothesis there is a three-party system practically identical in proportion to votes received.

NEP, Electoral Variation and Electoral Volatility

2.1.F. Effective number of parties (NEP_e), by Laakso and Taagepera (1979): among all the indexes mentioned in this article, the NEP has been the most adopted in Brazilian literature (KINZO, 2004; BRAGA, 2010; NICOLAU, 2005 and 2012). It would be no exaggeration to say that NEP has become the standard measure of party fragmentation (LIJPHART, 1999; COPPEDGE, 2001).

The NEP_e is another measure of dispersion and concentration of the party system derived from the HH index, but with an easier and more intuitive interpretation. The NEP_e expresses the number of parties of the same size that, in a hypothetical situation, would present the same fractionalization of the parties found in the reality.

If the percentages of votes received by the parties are equal, $NEP_e = N_e$. Aside from this extreme situation, $NEP_e < N_e$. The NEP_e can theoretically vary from 1 to infinity, depending on the number of parties competing in the election and the votes received. Mathematically, one has:

$NEP_e = 1 \div HH$. Or:

$NEP_e = 1 \div \sum V_i^2$, where "V" and "i" are as defined in 2.1.D.

According to the data used to calculate the F_e index in Table 4, in situation 1, $NEP_e = 1 \div 0,8152 = 1,227$. In situation 2, $NEP_e = 1 \div 0,3334 = 2,999$. It is seen that in these examples the result of the NEP_e is intuitive because in situation 1, there is an almost absolute party and the NEP_e is 1,227 and in situation 2, there are three parties of almost the same size and the NEP_e is 2,999.

Santos (2004) and Nicolau (2005) warn, however, that the "intuitive" adherence of the NEP_e to the party reality must be treated with care, since there is no way of starting from the index to find the most significant parties. In the scope of the discussion established between the authors, attention is drawn to the most adequate denomination of the index, that is, effective number of parties, and not number of effective parties, since the index does not allow, as said, to indicate which more important or significant parties of the reality experienced[10].

2.1.G. Average variation of party electoral participation (in successive elections) (E_e), by Rae (1967): is the comparison between the fractions of votes obtained by the parties in two elections, thus considering a same variable in two elections. So far, the measures considered only one variable in just one election, which is different in this case.

In the calculation of E_e, parties are ordered by name, in paired fashion, not by position in the election. So, they are compared according to their performance in the two elections. the measurements are taken in module. As it is an aggregate index, which considers all parties, the average of the variations is evaluated, whose result will be between zero and 100, meaning zero the absence of changes in the party system and 100, the maximum variation.

It is denoted as:

$$E_e = 1/n \sum_{i=1}^{n} |V_i - V_i'|, \text{ where:}$$

"n" = number of parties that make up the system;

"i" = each party that received votes in two subsequent elections;

"V_i" and "V_i'" = fraction of votes that the party i received in the two elections considered.

In the example of Table 7, the average change in votes was 20%, which shows some instability in the party system.

Table 7 – Example of average variation of parties' electoral participation (E_e)

[10] According to Braga (2010), the NEP_e in the Brazilian Chamber of Deputies elections, between 1982 and 2006, was, on average, 7,5.

	V_i	V_i'	Variation or $\lvert V_i - V_i' \rvert$
Party A	0,20	0,40	0,20
Party B	0,50	0,20	0,30
Party C	0,30	0,40	0,10
Σ	1,00	1,00	0,60
E_e			1/3 * 0,60 = 0,20

Source: adapted from Rae (1967, p. 59).

For the Brazilian case, one issue to consider is the recurrent creation, extinction and merger of party initials, which requires adaptations. Perhaps the index will make sense if only the traditional and larger parties are accompanied and encompass all others in a stable category of "Others".

The E_e allows the volatility of party electoral strength to be examined – as the volume of votes distributed to parties has changed over the course of the elections.

An inherent characteristic of this index is that, because it deals with aggregate votes, without identification of the voter, there can be great variation of positions between voters and they can be annulled, that is, the parties maintain the same fractions of total votes, but the voters who gave them support were different in each election. Another situation is that the electorate can change size, increase or decrease, and the party maintain the number of votes, but not the proportion of votes. In this case, the party may be very stable vis-à-vis its electorate, however, the index may point to movements. With these reservations, it can not be said that the index measures electoral change – which would be related to changes in the position of specific voters – but only the variation in the electoral strength of the parties, which is given by the relative volume of votes received. These criticisms are also developed for the next index.

2.1.H. Electoral Volatility (V_e), by Pedersen (1979): the index registers, by comparing two elections, the relative gains accumulated by all parties that increased their share of total votes or, symmetrically, the relative losses accumulated by the parties that had their electoral participation diminished[11]. Electoral volatility shows changes or similarities in the party system over time, which can be influenced by factors such as: changes in values in society, creation or extinction of parties and changes in the electoral system.

Pedersen's calculations of V_e, closely approximates that o E_e, by Rae – the difference

[11] Coppedge (2001) understands that volatility could also be calculated for any pair of elections. The author chooses the first and last election of a time interval to determine the value of the index, in a method that, according to him, better explains the cumulative variation of a long period of time than in the model of Pedersen (1979), that would account for "[...] subsequently incomplete variations" (Coppedge, 2001, p. 236, our translation). However, it must be said that in the calculation in which only two extreme moments are taken, the intermediate variations are lost, which may explain much about the partisan dynamics.

between them is basically at the moment of division. To get to V_e, add the rates of variation of votes of each party and divide by 2. Therefore,

$$V_e = \sum_{i=1}^{n} |V_i - V_i'| / 2,$$

where "V_i" and "V_i'" are defined as in 2. 1.G.

Gross change, without division, refers to the gains of some parties plus the losses of others, which are the two sides of the same coin. When dividing by 2 the gross result, the index gains better sense, intuitive, because it shows how much the parties have won or lost in terms of proportion of votes between two elections.

The result of V_e, presented in percentage, will be a value between zero and 100%. The closer to zero, the more stability in electoral competition and more institutionalization of the party system, because there is no substantive aggregate variation. The closer to 100%, the more instability and low institutionalization, because in its extreme, indicates that all parties existing in an election have lost all their votes in the consecutive election and are replaced by other parties. Adopting the same values in Table 7, $V_e = 60\% \div 2 = 30\%$, revealing still more instability in the party system than according to E_e. So, mathematically, E_e and V_e bring different information.

Electoral volatility has been widely used in the literature, with the intention of evaluating the degree of institutionalization of the Brazilian party system (KINZO, 2004; KINZO, 2005; BOHN and PAIVA, 2009; BRAGA, 2010)[12]. However, Nicolau (1997) emphasizes the need for care in interpreting this index, since it overlooks changes both in the number of voters in a country and in the individual preferences of voters. For example, if there are 100 more voters in election 2 than in election 1, some parties may win votes without other parties losing. In this case, a party supported by the same voters in two elections only stopped growing, which may not be clear when one appreciates its relative fall. By keeping voters from the first election, one might think of party stability rather than volatility, as the index suggests.

In another situation, if all voters of Party A in election 1 choose Party B in election 2, and vice versa, volatility will be zero, although there has been change in voter preferences. A possible and relevant problem for Brazil also concerns how to fit into the index the cases of creation, merger and extinction of parties[13].

12 To give you an idea, Brazil's average electoral volatility in elections to the Chamber of Deputies from 1990 to 2006 was 13.8% (Braga, 2010).

13 Wren and Mcelwain (2007) claim that there is controversy among authors at this point. Following researchers such as Lijphart, the authors choose, in the case of party fusion, to add the values of the separated parties in the election 1 and to compare that sum to the value of the new party in the election 2, disregarding the values referring to "leftovers", that is, extinguished parties without the creation of new parties, and the creation of new parties without the merger of preexisting parties. In the case of a party split, Mainwaring and Torcal (2005) decided to compare the value of the largest party resulting from the division in the election 2 with the value of the party not yet split in the election 1, and treat the value of the minor (s) party (s) in election 2 as not having received votes in election 1.

2.2 Measures with Seats

Measures related to parliamentary seats are, in their almost totality, identical to those of votes, only the variable that feeds the calculations changes. They are always indicated by capital letter (s) accompanied by subscript "p", which means parliamentary party.

2.2.A. Number of parliamentary parties (N_p), by Rae (1967): is the number of parties that won some parliamentary seat in a determined election. It portrays only the number of parties that obtained at least one seat in the election, not indicating the parliamentary strength of each party. Its calculation is the same as N_e, only seats are used instead of votes. The comparison between N_e and N_p allows us to gauge the degree to which the electoral system has penalized certain parties, that is, those who obtained votes, but not seats. It is a rule that some parties get votes, but not seats, making $N_e > N_p$.

2.2.B. Representation of the strongest parliamentary party (P_p), by Rae (1967): corresponds to the percentage of seats received by the largest parliamentary party. It emphasizes the strongest party in the parliamentary arena, although it does not incorporate the parliamentary force of the other parties. The comparison between P_e and P_p which usually happens in general. In an extreme case of $P_e < P_p$, which usually happens in general. In an extreme case of $P_e < 50\%$, and $P_p > 50\%$, there is a party that did not obtain most votes, but which obtains a parliamentary majority. This is called "manufactured fabricated" (Nicolau, 2012, p. 26) or "manufactured majority".

2.2.C. Representation of the two strongest parliamentary parties (W_p), by Rae (1967): the sum of the percentages of seats received by the two largest parliamentary parties. It shows how the composition of the Legislative House is concentrated in the two largest parties. However, it does not display the percentages of seats in each of the remaining parties. Similar reasoning can be applied to the previous index – when comparing W_e with W_p, one can see how much the system tends to favor or prejudice the two most voted parties.

2.2.D. Index of fractionalization of seats (F_p), by Rae (1967): but now seats and no more votes are computed. The comparison between F_e and F_p suggests the extent to which an electoral system fractionates or concentrates the distribution of political forces by converting votes into seats. In general, systems tend to have less fractionalization of seats than votes ($F_e > F_p$).

2.2.E. Effective number of parliamentary parties (NEP_p), by Laakso and Taagepera (1979): this is the same logic and the same calculations of the NEP_e, although here applied to seats. If the parties obtain an equal number of seats, $NEP_p = N_p$; if the parties do not obtain the same number of seats, $NEP_p < N_p$.

The comparison between NEP_p and NEP_e points to applications like the comparison of F_e and F_p – indicates the extent to which an electoral system fractionates or concentrates the distribution of political forces by converting votes into seats[14].

2.2.F. Minimum parliamentary majority (A), by Rae (1967): expresses the minimum number of parties necessary to form the majority coalition in the Legislature. The relevance of the index stems from the importance of majorities within the Legislative.

The parties are distributed in decreasing order of seats and the lowest number of parties is 50% or more. This variable is associated with the relative stability of governments and governability. It does not offer an analogy with the measures regarding the votes received by the parties. It does not offer an analogy with the measures regarding the votes received by the parties.

Table 8 – Example of the minimum parliamentary majority (A)

C	Situation 1	Situation 2
C_a	0,45	0,20
C_b	0,40	0,19
C_c	0,15	0,15
C_d	0,00	0,15
C_e	0,00	0,14
C_f	0,00	0,10
C_g	0,00	0,07
A	2	3

Source: Rae (1967, p. 63).

In situation 1, A is equal to two parties, and in situation 2, A is equal to three parties. It is noted that the measure accounts for the potential arrangements for governability in a parliament, but, as major parties may not be associated, this variable does not always represent the number of parties that will form the majority coalition – if a majority coalition will be constituted. It is worth adding that supermajorities are often built, or else minority coalitions to govern a country (the former far exceed 50% of the seats and the latter do not reach that level).

2.2.G. Average variation of parliamentary participation of parties (in successive elections) (E_p), by Rae (1967): compares the total number of seats per party, of all parties that

[14] According to Nicolau (2012), in the 2000s elections held until 2010, the NEP_p regarding the distribution of seats in the Brazilian Chamber of Deputies was 10,4.

obtained seats in an election (C_i), and the total number of seats of these parties in the immediately preceding election (C_i'). The form of calculation is like that of E_e, treated above, changing only the voting data for seats. The comparison between E_e and E_p allows us to measure the degree to which electoral laws increase or decrease the effects of electoral changes in the parliamentary scenario.

2.2.H. Parliamentary Volatility (V_p), by Pedersen (1979): specifies the aggregate rate of change of the party system, in terms of seats, between two subsequent elections. Its construction and interpretation are analogous to that of electoral volatility (V_e), now with reference to seats and no longer to votes. Thus, the V_p is defined by the sum of the rates of change of seats of each party, divided by 2.

Like the comparison between E_e and E_p, the comparison between V_e and V_p indicates the degree to which the oscillations in the variation of votes influence the variation in the number of seats.

2.3 Measures with Votes and Seats

One of the main concerns in the study of electoral systems is to assess the degree of approximation between the will of the voters and the constitution of the Legislative, to assess whether there is compatibility between the voters' manifestation in the ballots and the political forces constituted in the legislative assemblies. In other words, check the proportionality of the electoral system (GALLAGHER, 1991).

The concern makes sense, because there is no electoral system in force capable of perfectly transforming the percentage of votes in the same proportion of seats. There are logico-mathematical problems that impede the achievement of this goal, such as the number of elected per district, known as magnitude – in that the more elected per district, the more likely the result is proportional – and the number of parties competing – in which the more parties compete, the less chance the system will remain proportional (GALLAGHER, 1991; NICOLAU, 2012).

In addition, there is a disproportion deliberated by the formulators of the electoral system, who may seek not to maintain the proportionality of votes / seats, but to ensure the formation of stable governments. Major Electoral Systems of simple majority, for example, essentially seek to choose the most voted candidate, which ends up forcing the creation of majority governments. In them there is no greater concern with proportional votes / seats.

To evaluate the relationship between votes and seats, the disproportionality indexes of Loosemore and Hanby (1971 apud GALLAGHER, 1991), the one by Rae (1971 apud GALLAGHER, 1991) and the least squares of Gallagher (1991).

2.3.A. Index of disproportionality of Loosemore and Hanby (LH[15]) (1971 apud Gallagher, 1991): measures the difference between the percentages of votes and seats in a determined election. Disproportionality does not refer to the outcome of a party, but to that of the election, as a whole.

In an electoral dispute, there are parties that receive a greater percentage of seats than of votes, and there are others in the opposite situation, who win fewer seats than votes. Thus, the LH index first adds up the votes-seat differences in the module, so that the contrary signs do not cancel out, and, as in V_e, divides the result generated by 2. The logic of the division is to measure how much of seats the system gave more or less to the parties. That is, the disproportionality itself, because conferring seats more or less are the two sides of the same phenomenon. So,

$$LH = 1/2 \times \sum_{i=1}^{n} |V_i - C_i|,$$ where:

"V_i" = the proportion of votes received by party i;

"C_i" = the proportion of seats received by party i;

"n" = number of parties that received votes in the election;

"i" = each party that received votes in the election.

Table 9 provides examples of the calculations for the three disproportionality indexes studied.

Table 9 – Examples of disproportionality indexes (LH, RAE and MQ)[16]

Parties	Votes (%)	Seats (%)	LH	RAE	Least Squares
A	40,10	43,10	3,00	3,00	9,00
B	29,00	28,70	0,30	0,30	0,09
C	20,00	19,20	0,80	0,80	0,64
D	10,00	9,00	1,00	1,00	1,00
E	0,47	0,00	0,47	-	0,22
F	0,43	0,00	0,43	-	0,19
Total			6,00	5,10	11,14
Index value			3,00	1,28	2,36

Source: adapted from Gallagher (1991, 39).

15 The notation LH, in allusion to the surnames of the authors, is not present in other works. To differentiate this index of disproportionality from others, we chose this notation in this article.

16 In Table 9, we chose to work with the percentage notation, not decimal notation, since the indexes are more clearly expressed in this way. For the least squares index, the calculations must be done in percentage notation, under penalty of error.

For the situation outlined in Table 9, the LH disproportionality index has a value of 3%. As the calculations are performed in percentages, the values can range from zero to 100%. Zero would be when proportionality was perfect – each party gets in seats exactly what it received in proportion to votes. One hundred would be when the disproportionality was maximal, that is, the hypothetical case in which all the parties that received votes did not obtain seats, and when those that stopped seats did not obtain votes.

2.3.B. Disproportionality index by Rae (RAE) (apud GALLAGHER, 1991): measures not the disproportionality of the election, but the average disproportionality per party. The index excludes from the calculation the parties that obtained less than 0.5% of the votes.

$$RAE = 1/n \times \sum_{i=1}^{n} |V_i - C_i|$$, where "V_i", "C_i", "n" and "i" defined in 2.3.A.

The value of the RAE index is, as a rule, lower than that of LH, equating only if the system is two-party (Gallagher, 1991). It is important to emphasize that RAE denotes the average disproportionality for the parties, which is different and complementary to the information offered by the LH index, which gives the system disproportionality.

The next index, of least squares (MQ), according to Gallagher (1991), is one capable of minimizing the problems of LH and RAE, and presents, as a rule, an intermediate value to these two.

2.3.C Least squares (MQ) (Gallagher, 1991): measures the difference between the total of votes and of seats obtained by the parties in an election, like the LH. The difference is that the method of least squares, usual in statistics, aggregates the differences between values from its elevation squared, while the LH modulates such differences.

$$MQ = [1/2 \times \sum_{i=1}^{n} (V_i - C_i)^2]^{1/2}$$, where "V_i", "C_i", "n" and "i" defined as in 2.3.A

The differences between squared V_i and C_i re that they are summed by MQ. Then they are divided by 2, to have the same logic of LH, which is to evaluate the disproportionality of an election and not of the parties. Finally, the square root is applied to the result to return to the initial magnitude. Gallagher (1991) points out that the value of MQ is, as a rule, between LH and RAE. As the example in Table 9 shows, that is exactly what we can notice.

The merit of the disproportionality index is to relate the two variables in analysis, votes and seats, in a concise calculation only. However, for the analyst, it is important to consider that, unlike LH, the MQ sub-represents the importance of small parties, preferential targets of disproportionality. n the view of Lijphart (1999) and Nicolau (2012), by raising the differences between votes and seats squared, MQ presents, as in the F and NEP calculations, the tendency to overestimate the impact of the large parties and to underestimate the effects of small parties in accounting for disproportionality.

Another relevant point is that LH and RAE make calculations with the "raw" percentages, that is, without change. Thus, the interpretation is simple and intuitive, because a percentage is found as a result, be it the disproportionality of the system or the average for the parties. For example, by RAE it can be said that the disproportionality between votes and seats (from the example of Table 9) per party is 1.28%. This is easily understandable, since its meaning is that the parties gain or lose, on average, 1.28% of seats in relation to their participation in the total votes. On the other hand, the Gallagher Least Squares do not allow an interpretation of this nature, since the squared elevation transforms the magnitude of the calculations[17]. From Table 9, does Gallagher's 2.36 score mean what? It is not the result for the system, nor the average of the parties. So, what meaning?

Gallagher (1991) makes an important consideration applicable to Brazil, that the calculation of disproportionality should not be applied at the national level, but for each constituency, because the distortions in each of them can add up or cancel , with the final result being imprecise. Something still of greater impact occurs when the circumscriptions - in the Brazilian case, the states - do not have the same relation between votes and seats[18].

3 Conclusion

Political parties, party systems, and electoral systems are some of the themes most explored by Political Science. However, there is a need for consolidation efforts, especially for didactic purposes, of the applicable analytical measures. Learning and knowing how to use indexes of fragmentation and concentration of votes and seats, as well as those of disproportionality, allows the researcher not only to understand the terms of the debate, but also to contribute to its advancement.

Based on the best-known literature in the area, this article aimed to describe, in a didactic way, classic measures of fragmentation and concentration of votes and seats in the Legislative and also of disproportionality. PTo do so, some of the main indexes treated and their theoretical sources were: the index of fractionalization of votes and that of Rae's seats (1967); the effective number of parties and that of the parliamentary parties of Laakso and Taagepera (1979); the

[17] Technically the "problem" is in the use of the denominator "2" and its subsequent extraction of the square root. This problem of a "cleaner" mathematical calculation that achieves average results, between LH and RAE, brings this considerable loss, that of intuitiveness.

[18] Nicolau (2012) states that the MQ of the 2000s elections for the Brazilian Chamber of Deputies, up to 2010, was 2.5%.

electoral and parliamentary volatility of Pedersen (1979); and the indexes of disproportionality of Rao and least squares by Gallagher (1991).

In addition to the presentation of indexes, examples and comments on the results, this work was concerned with reflecting on the application of them to the Brazilian electoral and party systems, problems and related particularities. In this sense, it is necessary to look at some peculiarities of the Country, such as the sub and over representation of states in the legislative sphere; the electoral coalitions; the recurrent creation, extinguishment and merger of parties; and the different magnitudes of constituencies. Such precautions would allow what might be called a conscientious use of indexes.

Probably the biggest lesson from the joint assessment of indexes is that each of them contributes with different information, and that an analysis cannot be confined to just one of them and take it as a faithful reproduction of a broader and more complex political context. To encompass electoral, partisan, and legislative phenomena requires a diversity of methods and techniques, and the proper use of indexes is an important element to make up the analytical framework.

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